6.3 Investigating Rational Exponents $x^{\frac{1}{n}}$

Ready: 10 minute warm up

1. a) Use the fact that $\sqrt{9} = 3$ to complete the following.

$\sqrt{9} \times \sqrt{9}$

b) Use the law of exponents for multiplication to complete the following

$9^{\frac{1}{2}} \times 9^{\frac{1}{2}}$

c) compare the statements in a) and b). What other mathematical operation does the exponent $\frac{1}{2}$ seem to be equivalent to?

Warm-up Answers

2. Because $2^3 = 8$ (two cubed equals eight), we say that the cube root of 8 is 2, and we write $\sqrt[3]{8} = 2$

a) Use the fact that $\sqrt[3]{8} = 2$ to complete the following.

$\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8}$

b) use the law of exponents for multiplication to complete the following.

$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}}$

c) compare the statements in a) and b). What other mathematical operation does the exponent $\frac{1}{3}$ seem to be equivalent to?

3. Extend your reasoning to make a generalization about the meaning of $x^{\frac{1}{n}}$ (where $x > 0$ and n is a natural number)

6.3 Working With Rational Exponents

Exponent Law

Exponential form radical form

radical sign $\sqrt{\ }$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

index

index: indicates what root you want

Evaluate:

$8^{\frac{1}{3}}$ $9^{\frac{1}{2}}$

Extend the rule:

Ex 1: Evaluate

$125^{\frac{2}{3}}$

Does order matter?

method 1 method 2

Quotient of bases with Rational Exponents

Ex 2: Evaluate

Does order matter?

$\dfrac{50^{\frac{1}{2}}}{2^{\frac{1}{2}}}$

method 1 $\dfrac{\sqrt{50}}{\sqrt{2}}$

method 2 $\left(\dfrac{50}{2}\right)^{\frac{1}{2}}$

General Rule: $$\frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}} = \left(\frac{a}{b}\right)^{\frac{1}{2}} = \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Product of radicals

Ex 3: Evaluate

Does order matter?

$16^{\frac{1}{3}} \times 4^{\frac{1}{3}}$

method 1

$\sqrt[3]{16} \times \sqrt[3]{4}$

$= (2.52) \times (1.58)$

$= 4$

method 2

$(16 \times 4)^{\frac{1}{3}}$

$= (64)^{\frac{1}{3}}$

$= \sqrt[3]{64}$

$= 4$

Alternate Method: create like bases

$(4^2)^{\frac{1}{3}} \times 4^{\frac{1}{3}}$

$= 4^{\frac{2}{3}} \times 4^{\frac{1}{3}}$

$= 4^{\frac{2}{3}+\frac{1}{3}}$

$= 4^{\frac{3}{3}}$

$= 4$

General Rule: $a^{\frac{1}{n}} \times b^{\frac{1}{n}} \times c^{\frac{1}{n}} = \sqrt[n]{a} \times \sqrt[n]{b} \times \sqrt[n]{c} = \sqrt[n]{a \times b \times c}$

Practice:

$121^{\frac{1}{2}}$

$= \sqrt[2]{121}$

$= 11$

$27^{\frac{1}{3}} = \sqrt[3]{27}$

$= 3$

$64^{\frac{2}{3}} = \sqrt[3]{64^2}$

$= \sqrt[3]{4096}$

$= 16$

$(-8)^{\frac{1}{3}} = \sqrt[3]{-8}$

$= -2$

$32^{-\frac{1}{5}} = \frac{1}{\sqrt[5]{32}}$

$= \frac{1}{2}$

$-81^{\frac{3}{4}} = -(\sqrt[4]{81})^3$

$= -(3)^3$

$= -27$

$\frac{\sqrt{12}}{\sqrt{8} \times \sqrt{6}}$

$= \frac{\sqrt{12}}{\sqrt{8 \times 6}}$

$= \frac{\sqrt{12}}{\sqrt{48}}$

$= \sqrt{\frac{12}{48}}$

$= \sqrt{\frac{1}{4}}$

$= \frac{1}{2}$

Ex 4: Simplify then evaluate:

$\frac{\left(8^{\frac{1}{6}}\right)^7}{8^{\frac{1}{2}}\, 8^{\frac{1}{3}}} = \frac{8^{\frac{7}{6}}}{8^{\frac{1}{2}+\frac{1}{3}}} = \frac{8^{\frac{7}{6}}}{8^{\frac{3}{6}+\frac{2}{6}}} = \frac{8^{\frac{7}{6}}}{8^{\frac{5}{6}}} = 8^{\frac{7}{6}-\frac{5}{6}}$

$= 8^{\frac{2}{6}}$

$= 8^{\frac{1}{3}}$

Practice:

Simplify the following expressions:

a) $\frac{(m^5)^{-\frac{9}{5}}}{\left(m^{-\frac{3}{2}}\right)^4} = \frac{m^{-9}}{m^{-6}}$

$= m^{-9-(-6)}$

$= m^{-3}$

$= \frac{1}{m^3}$

b) $\frac{m^{\frac{1}{2}} \bullet m^{\frac{3}{2}}}{(m^{-2})^{\frac{1}{2}}} = \frac{m^{\frac{1}{2}+\frac{3}{2}}}{m^{-1}}$ $\frac{1}{2}+\frac{3}{2} = \frac{4}{2} = 2$

$= \frac{m^2}{m^{-1}}$

$= m^{2-(-1)}$

$= m^3$

Practice:

p 415 # 1, 4, 6,

[9 - 11, 15,17](def),

20a